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A priori error analysis of a fully-mixed finite element method for a two-dimensional fluid-solid interaction problem *

Carolina Domínguez[†] Gabriel N. Gatica[‡] Salim Meddahi[§] Ricardo Oyarzúa[¶]

Abstract

We introduce and analyze a fully-mixed finite element method for a fluid-solid interaction problem in 2D. The model consists of an elastic body which is subject to a given incident wave that travels in the fluid surrounding it. Actually, the fluid is supposed to occupy an annular region, and hence a Robin boundary condition imitating the behavior of the scattered field at infinity is imposed on its exterior boundary, which is located far from the obstacle. The media are governed by the elastodynamic and acoustic equations in time-harmonic regime, respectively, and the transmission conditions are given by the equilibrium of forces and the equality of the corresponding normal displacements. We first apply dual-mixed approaches in both domains, and then employ the governing equations to eliminate the displacement u of the solid and the pressure p of the fluid. In addition, since both transmission conditions become essential, they are enforced weakly by means of two suitable Lagrange multipliers. As a consequence, the Cauchy stress tensor and the rotation of the solid, together with the gradient of p and the traces of u and p on the boundary of the fluid, constitute the unknowns of the coupled problem. Next, we show that suitable decompositions of the spaces to which the stress and the gradient of p belong, allow the application of the Babuska-Brezzi theory and the Fredholm alternative for analyzing the solvability of the resulting continuous formulation. The unknowns of the solid and the fluid are then approximated by a conforming Galerkin scheme defined in terms of PEERS elements in the solid, Raviart-Thomas of lowest order in the fluid, and continuous piecewise linear functions on the boundary. Then, the analysis of the discrete method relies on a stable decomposition of the corresponding finite element spaces and also on a classical result on projection methods

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[†]Departamento de Ingeniería Matemática, Facultad de Ciencias Físicas y Matemáticas, Universidad de Concepción, Casilla 160-C, Concepción, Chile, e-mail: `cdominguez@ing-mat.udec.cl`

[‡]CI^{2}MA and Departamento de Ingeniería Matemática, Facultad de Ciencias Físicas y Matemáticas, Universidad de Concepción, Casilla 160-C, Concepción, Chile, e-mail: `ggatica@ing-mat.udec.cl`

[§]Departamento de Matemáticas, Facultad de Ciencias, Universidad de Oviedo, Calvo Sotelo s/n, Oviedo, España, e-mail: `salim@uniovi.es`

[¶]Departamento de Matemática, Facultad de Ciencias, Universidad del Bío-Bío, Casilla 3-C, Concepción, Chile, e-mail: `royarzua@ubiobio.cl`

for Fredholm operators of index zero. Finally, some numerical results illustrating the theory are presented.

Key words: mixed finite elements, Helmholtz equation, elastodynamic equation

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