LA SERENA NUMÉRICA I
Sexto Encuentro de Análisis Numérico de Ecuaciones Diferenciales Parciales
Departamento de Matemáticas, Universidad de La Serena, Diciembre 14–16, 2011

Analysis of a Modified Discontinuous Galerkin Recovery Scheme for Diffusion Problems

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Abstract

A theoretical error analysis using standard Sobolev space energy arguments is furnished for a class of discontinuous Galerkin (DG) schemes that are modified versions of one of those introduced by van Leer and Nomura. These schemes, which use discontinuous piecewise polynomials of degree q, are applied to a family of one-dimensional elliptic boundary value problems. The modifications to the original method include definition of a recovery flux function via a symmetric L^2-projection and the addition of a penalty or stabilization term. The method is found to have a convergence rate of $O(h^q)$ for the approximation of the first derivative and $O(h^{q+1})$ for the solution. Computational results for the original and modified DG recovery schemes are provided contrasting them as far as complexity and cost. Numerical examples are given which exhibit sub-optimal convergence rates when the stabilization terms are omitted.

Key words: Error Analysis, Discontinuous Galerkin, Recovery Scheme

Mathematics subject classifications (1991): 65N12, 65N30

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