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Solving Dirichlet boundary-value problems on general domains by extensions from subdomains

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Abstract

We present a technique for numerically solving Dirichlet boundary-value problems on a general domain Ω. We do not assume Ω polygonal. This is achieved by using suitably defined extensions from polyhedral subdomains $\mathsf{D}_h \subset \Omega$; the problem of dealing with curved boundaries is thus reduced to the evaluations of simple line integrals. The technique is independent of the representation of the boundary and of the space dimension. Moreover, it allows the use of only polyhedral elements and high order approximations. In the polyhedral subdomains D_h, we use a hybridizable discontinuous Galerkin method ([1]). We apply this technique to pure-diffusion ([2]) and convection-diffusion ([3]) problems and provide numerical experiments showing that the convergence properties of the resulting method are the same as those for the case in which $\Omega = \mathsf{D}_h$ whenever the distance of D_h to $\partial\Omega$ is of order h.

Key words: curved domains, immersed boundary methods, discontinuous Galerkin methods

References

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