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Finite volume schemes of high order for flows in different layers of unsaturated porous media*

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Abstract

Finite volume schemes of arbitrary order of accuracy in time and space for solving the flow in unsaturated porous media are presented. A high order WENO reconstruction procedure is applied to the cell averages at the current time level. The temporal evolution of the reconstruction polynomials is computed globally using the governing equations. A *global* space–time discontinous Galerkin (GDG) finite element scheme is used to resolve the small gradients produced by the flows in unsatured porous media. An iterative solution of the matrix procedure is used to solve the system of equation that results from the global scheme proposed. Finally an explicit space–time integration over each control volume, using the GDG solution at the Gaussian integration point allows the calculation of the intercell fluxes. The convergence of the new scheme developed is tested with the exact transient diffusion–reaction solution. Three test cases are presented for comparing the accuracy of our ADER-FV method of second to fourth order solution with the exact solution and a different finite elements scheme.

Key words: unsatured porous media, high–order schemes, finite volume methods, Richards equations, ADER schemes, space–time discontinous Galerkin.

Mathematics subject classifications (1991): 65N30, 65N12, 65N15, 74F10, 74B05, 35J05

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