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# LA SERENA NUMÉRICA I

Sexto Encuentro de Análisis Numérico de Ecuaciones Diferenciales Parciales

Departamento de Matemáticas, Universidad de La Serena, Diciembre 14–16, 2011

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## Implicit-Explicit Runge–Kutta schemes and finite elements with symmetric stabilization for advection-diffusion equations

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### Abstract

We analyze a two-stage explicit-implicit Runge–Kutta scheme for time discretization of advection-diffusion equations. Space discretization uses continuous, piecewise affine finite elements with interelement gradient jump penalty; discontinuous Galerkin methods can be considered as well. The advective and stabilization operators are treated explicitly, whereas the diffusion operator is treated implicitly. Our analysis hinges on  $L^2$ -energy estimates on discrete functions in physical space. Our main results are stability and quasi-optimal error estimates for smooth solutions under a standard hyperbolic CFL restriction on the time step, both in the advection-dominated and in the diffusion-dominated regimes. The theory is illustrated by numerical examples.

### References

- [1] E. Burman and A. Ern, Implicit-Explicit Runge–Kutta schemes and finite elements with symmetric stabilization for advection–diffusion equations, *ESAIM Math. Mod. Numer. Anal.*, To appear.

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