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Tridiagonal preconditioning for Poisson-like difference equations with flat grids: Application to incompressible atmospheric flow problem

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Abstract

The convergence of many iterative procedures, in particular of the conjugate gradient method, strongly depends on the condition number of the linear system to be solved. In cases with a large condition number, therefore, preconditioning is often used to transform the system into an equivalent one, with a smaller condition number and therefore faster convergence. For Poisson-like difference equations with flat grids, the vertical part of the difference operator is dominant and tridiagonal and can be used for preconditioning. Such a procedure has been applied on incompressible atmospheric flows to maintain incompressibility, where a system of Poisson-like difference equations is to be solved for the dynamic pressure part. In the mesoscale atmospheric model KAMM, convergence has been speeded up considerably by tridiagonal preconditioning, even though the system matrix is not symmetric and, hence, the biconjugate gradient method must be used.

Key words: Poisson-like equation; condition number; preconditioning; convergence acceleration; atmospheric model; flat grids.

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