

---

# LA SERENA NUMÉRICA I

Sexto Encuentro de Análisis Numérico de Ecuaciones Diferenciales Parciales

Departamento de Matemáticas, Universidad de La Serena, Diciembre 14–16, 2011

---

## Numerical Solution of an Equation Describing the Centrifugal Settling with Coalescence of Polydisperse Liquid-Liquid Dispersions Using the Fixed Pivot Technique \*

ANTONIO GARCÍA A.<sup>†</sup>

### Abstract

The centrifugal settling and coalescence of a polydisperse dispersion of two immiscible liquids with a continuous droplet size distribution in a rotating tube or basket centrifuge can be modeled by a integro-partial differential equation (IPDE), which is an extension of a previous spatially distributed population balance equation describing gravity settling and coalescence of such a mixture. This IPDE is projected onto a system of convective dominant partial differential equations by discretizing the droplet diameter. This is accomplished by using the fixed-pivot technique of Kumar and Ramkrishna (Chem. Eng. Sci. 51 (1996a) 1311–1332) handling any two integral properties of the population number density. The resulting system of PDEs is split into two systems, of homogeneous PDEs and ODEs. The homogeneous PDEs and the ODEs are discretized using the second-order non-oscillatory central differencing scheme of Kurganov and Tadmor (J. Comput. Phys. 160 (2000) 241–282) and the second-order two-stage Runge-Kutta method, respectively. Simulations are presented, illustrating the coalescence and the formation of sediment of the disperse phase, and the effect of various centrifuge geometries for both cases, when the disperse phase (droplets) is less dense than the continuous phase and viceversa. In particular, the model predicts the radial variation of the composition of the disperse phase layer forming at the inner or outer wall.

**Key words:** centrifugal settling, coalescence, polydisperse dispersion, population balance equation

### References

- [1] ATTARAKIH, M.M., BART, H.-J. AND FAQIR, N.M., *Numerical solution of the spatially distributed population balance equation describing the hydrodynamics of interacting liquid-liquid dispersions*. Chemical Engineering Science, vol. 59, pp. 2567-2592, (2004).

---

\*This research was supported by Fondecyt Project 11085069, and by Centro de Investigación Científica y Tecnológica para la Minería, CICITEM.

<sup>†</sup>Departamento de Ingeniería Metalúrgica, Universidad Católica del Norte and CICITEM, Antofagasta, Chile, e-mail: [agarcia@ucn.cl](mailto:agarcia@ucn.cl)

- [2] BERRES, S. AND BÜRGER, R., *On gravity and centrifugal settling of polydisperse suspensions forming compressible sediments*. International Journal of Solids and Structures, vol. 40, pp. 4965-4987, (2003).
- [3] BÜRGER, R., GARCÍA, A., KARLSEN, K.H. AND TOWERS, J.D., *A kinematic model of continuous separation and classification of polydisperse suspensions*. Computers and Chemical Engineering, vol. 32, pp. 1181-1202, (2008).
- [4] KUMAR, A. AND HARTLAND, S., *Gravity Settling in Liquid/Liquid Dispersions*. The Canadian Journal of Chemical Engineering, Vol. 63, pp. 368-376, (1985).
- [5] KUMAR, S. AND RAMKRISHNA, D., *On the solution of population balance equations by discretization - I. A fixed pivot technique*. Chemical Engineering Science, vol. 51, pp. 1311-1332, (1996).
- [6] KURGANOV, A. AND TADMOR, E., *New high resolution central schemes for nonlinear conservation laws and convection-diffusion equations*. Journal of Computational Physics, vol. 160, pp. 241-282, (2000).
- [7] LEVEQUE, R.J., *Finite Volume Methods for Hyperbolic Problems*. Cambridge University Press: New York, (2002).
- [8] MASLIYAH, J.H., *Hindered settling in a multiple-species particle system*. Chemical Engineering Science, vol. 34, pp. 1166-1168, (1979).
- [9] PADILLA, R., RUIZ, M.C. AND TRUJILLO, W., *Separation of liquid-liquid dispersions in a deep-layer gravity settler: Part I. Experimental study of the separation process*. Hydrometallurgy, vol. 42, pp. 267-279, (1996).
- [10] HULBURT, H. AND KATZ, S., *Some problems in particle technology. A statistical mechanical formulation*. Chemical Engineering Science, vol. 19, pp. 555-574, (1964).
- [11] ZHANG, X., WANG, H. AND DAVIS, R.H., *Collective Effects of Temperature Gradients and Gravity on Droplet Coalescence*. Physics of Fluids A, vol. 5, pp. 1602-1613, (1993).