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Analysis of a Cartesian PML approximation to acoustic scattering problems in $\mathbb{R}^n$

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Abstract

We consider the application of a perfectly matched layer (PML) technique in Cartesian geometry to approximate solutions of the acoustic scattering problem in the frequency domain. Our goal is to develop stability and exponential convergence for PML approximations on truncated (bounded) domains. To illustrate the issues, we start by examining the analysis for a simple PML problem in one spatial dimension. We then consider some of the issues in developing analogous stability properties for multidimensional PML approximations. In particular, we consider the case when the PML stretching is applied independently in each coordinate direction, the so-called Cartesian PML. The compact perturbation arguments used in the one dimensional problem fail in higher dimensions. Two distinct approaches circumventing this problem will be described. The first is based on locating the essential spectrum of the PML operator. The second involves the development of a solution operator for the PML equations in terms of an integral transform. The second approach is particularly interesting as it leads to estimates which show that stability of the truncated PML problem can be achieved provided that the product of the size of the truncated domain times the PML strength is sufficiently large. This means that stability can be achieved with a fixed sized computational domain in conjunction with a large PML coefficient. Finally, the results of computational experiments illustrating the theory will be given and extensions to other problems will be discussed.

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