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A strong finite element coupling of fluid flow with porous media flow

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Abstract

Our aim is to propose a new finite element scheme for the coupling of fluid flow with porous media flow. In the approach in which the Darcy problem is set in its natural $H(\mathrm{div})$ formulation (cf. [1] and the references therein) and the Stokes problem is expressed in velocity-pressure form, the transmission condition ensuring global mass conservation is handled weakly through a Lagrange multiplier representing the pressure on the coupling interface Σ. This strategy requires two finite element meshes on the coupling boundary Σ satisfying a stability condition between their corresponding mesh sizes. Such a restriction is quite cumbersome in 3D computations.

We propose here a formulation in which the balance on Σ between the Darcy flux and the normal component of the fluid velocity is imposed exactly (strongly) at the discrete level. We discuss the kind of conditions the discrete spaces for Stokes and Darcy have to satisfy to deliver stable methods for the global formulation.

Key words: mixed finite elements, Stokes problem, Darcy problem

Mathematics subject classifications (1991): 65N30, 65N12, 65N15, 74F10, 74B05, 35J05

References

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