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Numerical analysis of a Stokes' approximation for hydrodynamic interactions of droplets in a turbulent tropical cloud *

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Abstract

The numerical simulation of tropical clouds is a challenging problem requiring the numerical solution of the Navier-Stokes equations. In this work we focus on the hydrodynamic interactions of droplets. The typical droplet size is smaller than the smallest length scale in the turbulent simulation so the Stokes' equation is used [9, 10, 1]. The Stokes' approximation requires a solution of a linear system of equation every time the droplets move, thus a fast and accurate method is needed. The linear system is solved by GMRes [4, 5, 8] and a Restricted Schwarz preconditioner [2]. We explore the convergence properties of the sequence of linear system. A perturbation analysis reveals key features of why GMRes is so effective in this case [3, 5, 6]. We finally discuss a parallel implementation on massively parallel peta-scale machines [7] and a possible extension to use a uniformly valid approximation for the interactions.

Key words: Stokes' approximation, convergence of GMRes, parallel computing

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