LA SERENA NUMÉRICA I
Sexto Encuentro de Análisis Numérico de Ecuaciones Diferenciales Parciales
Departamento de Matemáticas, Universidad de La Serena, Diciembre 14–16, 2011

Sedimentation of suspensions: new aspects of an old problem*

Raimund Bürger†

Abstract

The sedimentation of suspensions consisting of small solid particles dispersed in a viscous fluid is a fundamental process occurring in nature and many industrial applications. Of particular importance to copper mining in Chile are so-called clarifier-thickeners, which are large-diameter, continuously operated settling tanks in which tailings arising from a flotation process are allowed to settle. On the other hand, applications in mineral processing and other areas frequently involve so-called polydisperse suspensions, in which the solid particles belong to a number N of species differing in size or density. The species usually segregate and form areas of different composition. It is the purpose of this contribution to present recent advances in the development of numerical methods for the simulation of both applications.

We start with a general introduction to sedimentation models and then consider a 1-d model of polydisperse sedimentation, which is given by a system of N strongly coupled, nonlinear first-order conservation laws for the N local solids volume fraction of the solids species [1, 2]. Solutions of this system are usually discontinuous with kinematic shocks separating areas of different composition. It is demonstrated how the so-called secular equation (can be utilized to estimate the region of hyperbolicity and to extract the eigenstructure of the Jacobian with acceptable effort. This information can be employed for the implementation of weighted essentially non-oscillatory (WENO) schemes for the accurate numerical approximation of the model. Numerical examples are presented.

Multidimensional versions of the same model involve the necessity to solve additional equations for the motion of the mixture, e.g. the Stokes system with a concentration-dependent viscosity and forcing term. We demonstrate how multiresolution methods may be applied to the numerical solution of such coupled problems for the case of settling of suspensions in inclined channels [4].

Finally, we consider a multi-dimensional sedimentation-consolidation model in clarifier-thickener units [3]. The model is given by a parabolic equation describing the evolution of the local solids concentration coupled with a version of the Stokes system for an incompressible fluid. In cylindrical coordinates, and if an axially symmetric solution is assumed, the original problem reduces to two space dimensions. A novel finite volume

*This research was partially supported by FONDAP and BASAL projects CMM, Universidad de Chile, Centro de Investigación en Ingeniería Matemática (CI^{2}MA), Universidad de Concepción, and Fondecyt 1090456.

†**CI^{2}MA** and Departamento de Ingeniería Matemática, Facultad de Ciencias Físicas y Matemáticas, Universidad de Concepción, Casilla 160-C, Concepción, Chile, e-mail: `rburger@ing-mat.udec.cl`

element method is introduced for the spatial discretization, where the velocity field and the solids concentration are discretized on two different dual meshes. Numerical experiments illustrate properties of the model and the satisfactory performance of the proposed method.

This presentation is based on joint work with Rosa Donat and Pep Mulet (Valencia, Spain), Carlos A. Vega (Barranquilla, Colombia), Ricardo Ruiz-Baier (Lausanne, Switzerland), Kai Schneider (Marseille, France) and Héctor Torres (La Serena, Chile).

References

[1] R. Bürger, R. Donat, P. Mulet and C.A. Vega, Hyperbolicity analysis of polydisperse sedimentation models via a secular equation for the flux Jacobian, SIAM J. Appl. Math. **70** (2010), 2186–2213.

[2] R. Bürger, R. Donat, P. Mulet and C.A. Vega, On the implementation of WENO schemes for a class of polydisperse sedimentation models, J. Comput. Phys. **230** (2011), 2322–2344.

[3] R. Bürger, R. Ruiz-Baier and H. Torres, A stabilized finite volume element method for sedimentation-consolidation processes. Preprint 2011-15, Departamento de Ingeniería Matemática, Universidad de Concepción; submitted.

[4] R. Bürger, R. Ruiz-Baier, K. Schneider and H. Torres, A multiresolution method for the simulation of sedimentation in inclined channels, Int. J. Numer. Anal. Model., to appear.