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Greedy Approximation of a High-Dimensional PDE arising from Kinetic Theory Models of Dilute Polymers*

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Abstract

We investigate the convergence of a nonlinear approximation method introduced by Ammar et al. for the numerical solution of high-dimensional Fokker–Planck equations featuring in Navier–Stokes–Fokker–Planck systems that arise in kinetic models of dilute polymers. In the case of Poisson's equation on a rectangular domain in $\mathbb{R}^2$, subject to a homogeneous Dirichlet boundary condition, the mathematical analysis of the algorithm was carried out recently by Le Bris, Lelièvre and Maday by exploiting its connection to greedy algorithms from nonlinear approximation theory explored, for example, by DeVore and Temlyakov; hence, the variational version of the algorithm, based on the minimization of a sequence of Dirichlet energies, was shown to converge. We extend the convergence analysis of the pure greedy and orthogonal greedy algorithms considered by Le Bris, Lelièvre and Maday to the technically more complicated case where the Laplace operator is replaced by a high-dimensional Ornstein–Uhlenbeck operator with unbounded drift, of the kind that appears in Fokker–Planck equations that arise in bead-spring chain type kinetic polymer models with finitely extensible nonlinear elastic potentials, posed on a high-dimensional Cartesian product configuration space $\mathsf{D} = D_1 \times \cdots \times D_N$ contained in $\mathbb{R}^{Nd}$, where each set D_i, $i = 1, \dots, N$, is a bounded open ball in $\mathbb{R}^d$, $d = 2, 3$.

Key words: Nonlinear approximation, greedy algorithm, Fokker–Planck equation

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