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A Particular Mixed Formulation for Interface Approximation of Darcy Flow in a Narrow Channel *

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Abstract

A particular mixed formulation is introduced for the singular problem of Darcy flow in a porous medium in a region containing a narrow fracture or channel with width $\mathcal{O}(\epsilon)$ and high permeability $\mathcal{O}(1/\epsilon)$. The formulation allows to introduce more general transmission conditions for fluid exchange between the domains than those allowed by the L^2-H^1 and the $H(\text{div})$-L^2 mixed formulations. For a channel defined by two vertical translates of a piecewise C^2 surface the solution converges as $\epsilon \to 0$ to that of Darcy flow coupled to tangential flow on the lower-dimensional interface or boundary. Numerical experiments will be presented to illustrate aspects such as convergence, stability and implementation of the finite element method for the interaction between a regular domain in $\mathbb{R}^N$ and a lower-dimensional manifold.

Key words: porous media, heterogenous, fissures, coupled Darcy systems.

Mathematics subject classifications (1991): Primary 35K50, 35B25; Secondary 80A20, 35F15

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