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Polynomial decay for transmission problem in viscoelasticity*

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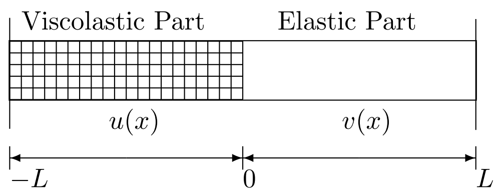
Abstract

We study the asymptotic behavior of solutions for a transmission problem with localized viscoelasticity of Kelvin Voigt type. In particular, we are interested in the following system of hyperbolic equations with boundary and transmission conditions

$$
\begin{aligned}
\rho_1\, u_{tt} - \kappa_1\, u_{xx} - \kappa_2\, u_{xxt} &= 0, && \text{in } \;]-L,\, 0[\times]0,\, \infty[\\
\rho_2\, v_{tt} - \kappa_3\, v_{xx} &= 0, && \text{in } \;]0,\, L[\times]0,\, \infty[, \\
u(-L,\, t) &= 0, && v(L,\, t) = 0, \qquad t \geq 0 \\
u(0,\, t) &= v(0,\, t), && \kappa_1\, u_x(0,\, t) + \kappa_2\, u_{xt}(0,\, t) = \kappa_3\, v_x(0,\, t)
\end{aligned}
$$

where the functions $u = u(x,\, t)$ and $v = v(x,\, t)$ represents the fraction field of a constituent (see figure on the right). The transmission problem has been of interest to many works, for instance, in the one-dimensional thermoelastic composite case, we can refer [1, 2]. We use the semigroup theory approach to show the well-posedness of the system. Additionally assuming regular enough initial conditions, we show the polynomial decay of the energy of the solution on $1/(1+t^2)$. The rate of decay si optimal and we consider the characterization due to Borichev and Tomilov [3] and the techniques developed by Liu and Rao [4] to prove it. On the other hand, a series of numerical examples highlight this asymptotic behavior.



Key words: Polynomial decay, transmission problem, C_0-semigroup, Numerical methods.

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