LA SERENA NUMÉRICA I

Sexto Encuentro de Análisis Numérico de Ecuaciones Diferenciales Parciales

Departamento de Matemáticas, Universidad de La Serena
La Serena, Diciembre 14 - 16, 2011

PROGRAMA y RESÚMENES

Índice

1. Introducción

El **Sexto Encuentro de Análisis Numérico de Ecuaciones Diferenciales Parciales** ha sido organizado en conferencias secuenciales de **45 y 30 minutos** de duración (40 y 25 minutos de exposición, respectivamente, y 5 minutos para preguntas y comentarios). Todas las charlas se llevarán a cabo en el Salon Elqui del **Centro Vacacional La Serena**.

En las páginas siguientes se detalla la programación correspondiente. Cuando hay más de un autor, aquel que aparece subrayado corresponde al expositor.

Los organizadores expresamos nuestro agradecimiento a los auspiciadores que se indican a continuación, los cuales han aportado gran parte de los recursos necesarios para el financiamiento de este evento:

- Departamento de Matemáticas, Universidad de La Serena,
- Centro de Modelamiento Matemático (CMM) de la Universidad de Chile,
- Facultad de Matemáticas de la Pontificia Universidad Católica de Chile, y
- Centro de Investigación en Ingeniería Matemática (CI^2MA) de la Universidad de Concepción.

Igualmente, extendemos nuestro reconocimiento y gratitud a todos los expositores, quienes han hecho posible la realización de **La Serena Numérica I**.

Comité Organizador

La Serena, Diciembre 2011

2. Miércoles, 14 de Diciembre

8.30-9.15 INSCRIPCIÓN

9.15-9.30 PALABRAS DE BIENVENIDA

[*Moderador*: N. HEUER]

9.30-10.15 Erik Burman, Alexandre Ern: *Implicit-Explicit Runge–Kutta schemes and finite elements with symmetric stabilization for advection-diffusion equations.*

10.15-10.45 Leonardo E. Figueroa, Endre Süli: *Greedy approximation of a high-dimensional PDE arising from kinetic theory models of dilute polymers.*

10.45-11.15 COFFEE BREAK

11.15-11.45 Nelson Moraga, Mauricio Godoy, Carlos Garrido: *On mathematical models and FVM simulations for convective solidification industrial processes.*

11.45-12.15 Roberto C. Cabrales, F. Guillén-González, J.V. Gutiérrez-Santacreu: *Stability and convergence for a complete model of mass diffusion.*

12.15-12.45 Ana M. Alonso Rodríguez: *Domain decomposition methods for eddy current problems.*

12.45-15.00 ALMUERZO

[*Moderador*: A. ALONSO-RODRÍGUEZ]

15.00-15.45 Joseph E. Pasciak: *Analysis of a cartesian PML approximation to acoustic scattering problems in* $\mathbb{R}^n$.

15.45-16.15 Alfredo Bermúdez, Bibiana López-Rodríguez, Rodolfo Rodríguez, Pilar Salgado: *Equivalence between two finite element methods for the eddy current problem.*

16.15-16.45 Carlos Acosta, Carlos E. Mejía: *Space marching and discrete mollification for nonlinear diffusion coefficient identification.*

16.45-17.15 COFFEE BREAK

17.15-17.45 Francisco Guillén González, Giordano Tierra: *On numerical schemes for a Cahn Hilliard diffuse interface model.*

17.45-18.15 Fernando Morales, R. E. Showalter: *A particular mixed formulation for interface approximation of Darcy flow in a narrow channel.*

18.15-18.45 Gabriel N. Gatica, Antonio Márquez, Salim Meddahi: *Analysis of an augmented fully-mixed finite element method for a three-dimensional fluid-solid interaction problem.*

19.30 COCKTAIL DE BIENVENIDA

3. Jueves, 15 de Diciembre

[*Moderador*: H. TORRES]

9.30-10.15 Salim Meddahi: *A strong finite element coupling of fluid flow with porous media flow.*

10.15-10.45 Ana Alonso Rodríguez, Jessika Camaño, Alberto Valli: *Inverse source problems for eddy current equations.*

10.45-11.15 COFFEE BREAK

11.15-11.45 Margareth Alves, Jaime Muñoz Rivera, Mauricio Sepúlveda, Octavio Vera, María Zegarra Garay: *Polynomial decay for transmission problem in viscoelasticity.*

11.45-12.15 Antonio García: *Numerical solution of an equation describing the centrifugal settling with coalescence of polydisperse liquid-liquid dispersions using the fixed pivot technique.*

12.15-12.45 Carlos F. Jerez-Hanckes, Carolina Urzúa: *Operator preconditioning for two-dimensional screen and fracture problems using boundary elements.*

12.45-15.00 ALMUERZO

[*Moderador*: C. JEREZ-HANCKES]

15.00-15.45 Raimund Bürger: *Sedimentation of suspensions: new aspects of an old problem.*

15.45-16.15 Carlos Zambra, Michael Dumbser, Eleuterio Toro, Nelson Moraga: *Finite volume schemes of high order for flows in different layers of unsaturated porous media.*

16.15-16.45 Carolina Domínguez, Gabriel N. Gatica, Salim Meddahi, Ricardo Oyarzúa: *A priori error analysis of a fully-mixed finite element method for a two-dimensional fluid-solid interaction problem.*

16.45-17.15 COFFEE BREAK

17.15-17.45 Catalina Domínguez, E. P. Stephan, M. Maischak: *Finite element and boundary element coupling for fluid-structure interaction.*

17.45-18.15 Mauricio Osorio, Donald French, Marshall Galbraith: *Analysis of a modified discontinuous Galerkin recovery scheme for diffusion problems.*

18.15-18.45 Leszek Demkowicz, Norbert Heuer: *Robust DPG method for convection-dominated diffusion problems.*

20.30 CENA DE CAMARADERÍA

4. Viernes, 16 de Diciembre

[*Moderador*: L. FIGUEROA]

9.30-10.15 Gernot Beer, Jerzey Rojek, Gerasimos Karlis, Lukasz Malinowski: *Adaptive coupling of different numerical methods.*

10.15-10.45 Melitta Fiebig-Wittmaack, Wolfgang Boersch-Supan, Ingeborg Bischoff-Gauss: *Tridiagonal preconditioning for Poisson-like difference equations with flat grids: Application to incompressible atmospheric flow problem.*

10.45-11.15 COFFEE BREAK

11.15-11.45 Alfredo Bermúdez, M. Dolores Gómez, Rodolfo Rodriguez, Pablo Venegas: *Numerical solution of transient nonlinear axisymmetric eddy current models with hysteresis.*

11.45-12.15 Leszek Demkowicz, Jay Gopalakrishnan, Ignacio Muga: *An update of the DPG method for wave propagation problems.*

12.15-12.45 Bernardo Cockburn, Manuel Solano: *Solving Dirichlet boundary-value problems on general domains by extensions from subdomains.*

12.45-15.00 ALMUERZO

[*Moderador*: G. GATICA]

15.00-15.45 J. Schöberl: Title to be announced.

15.45-16.15 Louis F. Rossi, Claudio E. Torres: *Numerical analysis of a Stokes' approximation for hydrodynamic interactions of droplets in a turbulent tropical cloud.*

16.15-16.45 COFFEE BREAK

16.45-17.15 R. Araya, Abner H. Poza, Frédéric Valentin: *An adaptive residual local projection finite element method for Navier–Stokes equations.*

17.15-17.45 Raimund Bürger, Kenneth H. Karlsen, Héctor Torres, John D. Towers: *A clarifier-thickener model including flocculant transport and adsorption.*

5. Resúmenes

LA SERENA NUMÉRICA I
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Space marching and discrete mollification for nonlinear diffusion coefficient identification *

Carlos D. Acosta[†] Carlos E. Mejía[‡]

Abstract

The discrete mollification method is a convolution-based filtering procedure suitable for the regularization of ill-posed problems. Combined with explicit space-marching finite difference schemes, it provides stability and convergence for a variety of coefficient identification problems in linear parabolic equations. In this work we extend such a technique to identify some nonlinear diffusion coefficients depending on an unknown space dependent function in one dimensional parabolic models. For the coefficients recovery process we present detailed error estimates and to illustrate the performance of the algorithms, several numerical examples are included.

Key words: mollification, parameter identification, space marching

Mathematics subject classifications (2010): 65N21, 65N12, 65N06, 80A23

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LA SERENA NUMÉRICA I
Sexto Encuentro de Análisis Numérico de Ecuaciones Diferenciales Parciales
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Domain decomposition methods for eddy current problems

ANA M. ALONSO RODRÍGUEZ *

Abstract

The usual setting for an eddy current problem distinguish between a conductive region and a non conductive air region. For this reason is natural to use domain decomposition methods for the numerical approximation of the solution. The aim of this talk is to review some two-domains formulations of the eddy current model in electromagnetism and their finite elements approximation. Particular attention will be pay to the case of non simply connected conductors. I analyze formulations that consider as main unknown the same field in both domains and also hybrid formulation that use different unknowns in the conductor and the insulator.

Key words: domain decomposition methods, eddy current problems, finite elements.

Mathematics subject classifications (2010): 65N30, 65N55.

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LA SERENA NUMÉRICA I
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Inverse source problems for eddy current equations *

Ana Alonso Rodríguez† Jessika Camaño‡ Alberto Valli §

Abstract

We study the inverse source problem for the eddy current approximation of Maxwell equations. As for the full system of Maxwell equations, ([1, 2, 3]) we show that a volume current source cannot be uniquely identified by the knowledge of the tangential components of the electromagnetic fields on the boundary, and we characterize the space of non-radiating sources. On the other hand, we prove that the inverse source problem has a unique solution if the source is supported on the boundary of a subdomain or if it is the sum of a finite number of dipoles. We address the applicability of this result for the localization of brain activity from electroencephalography and magnetoencephalography measurements.

Key words: eddy current problem, inverse problem.

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LA SERENA NUMÉRICA I

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Polynomial decay for transmission problem in viscoelasticity *

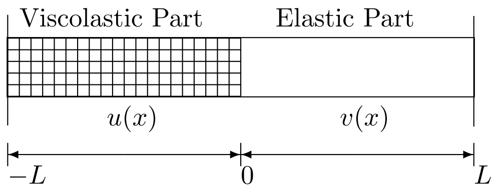
Margareth Alves† Jaime Muñoz Rivera‡ Mauricio Sepúlveda§
Octavio Vera¶ María Zegarra Garay‖

Abstract

We study the asymptotic behavior of solutions for a transmission problem with localized viscoelasticity of Kelvin Voigt type. In particular, we are interested in the following system of hyperbolic equations with boundary and transmission conditions

$$
\begin{aligned}
\rho_1\, u_{tt} - \kappa_1\, u_{xx} - \kappa_2\, u_{xxt} = 0, &\qquad \text{in} \quad]-L,\, 0[\times]0,\, \infty[\\
\rho_2\, v_{tt} - \kappa_3\, v_{xx} = 0, &\qquad \text{in} \quad]0,\, L[\times]0,\, \infty[, \\
u(-L,\, t) = 0, &\qquad v(L,\, t) = 0, \qquad t \geq 0 \\
u(0,\, t) = v(0,\, t), &\qquad \kappa_1\, u_x(0,\, t) + \kappa_2\, u_{xt}(0,\, t) = \kappa_3\, v_x(0,\, t)
\end{aligned}
$$

where the functions $u = u(x,\, t)$ and $v = v(x,\, t)$ represents the fraction field of a constituent (see figure on the right). The transmission problem has been of interest to many works, for instance, in the one-dimensional thermoelastic composite case, we can refer [1, 2]. We use the semigroup theory approach to show the well-posedness of the system. Additionally assuming regular enough initial conditions, we show the polynomial decay of the energy of the solution on $1/(1+t^2)$. The rate of decay si optimal and we consider the characterization due to Borichev and Tomilov [3] and the techniques developed by Liu and Rao [4] to prove it. On the other hand, a series of numerical examples highlight this asymptotic behavior.



Key words: Polynomial decay, transmission problem, C_0-semigroup, Numerical methods.

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LA SERENA NUMÉRICA I
Sexto Encuentro de Análisis Numérico de Ecuaciones Diferenciales Parciales
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An adaptive residual local projection finite element method for Navier–Stokes equations *

Rodolfo Araya† Abner H. Poza‡ Frédéric Valentin§

Abstract
An adaptive finite element scheme for the steady incompressible Navier–Stokes model is introduced and analyzed. This scheme is based on an RELP stabilized finite element method combined with a residual error estimator. In particular, the error estimator is analyzed theoretically and numerically through several well–established benchmarks.

Key words: stabilized finite element, Navier–Stokes equations, a posteriori error estimators

Mathematics subject classifications (2010): 65N30, 65N12, 65N15

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LA SERENA NUMÉRICA I
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Departamento de Matemáticas, Universidad de La Serena, Diciembre 14–16, 2011

Adaptive coupling of different numerical methods*

Gernot Beer† Jerzey Rojek‡
Gerasimos Karlis§ Lukasz Malinowski¶

Abstract

In the simulation of problems in engineering different material behavior may occur in different parts of the analyzed domain. The behavior may range from continuum to dis-continuum behavior. Of the three main numerical methods used for the simulation some are better than others for modeling different physical phenomena. The Boundary Element Method (BEM) for example is best suited for elastic continuum problems, the Finite Element Method (FEM) has been found to work well for non-linear material problems and finally the Discrete Element Method (DEM) is ideally suited for modeling discontinuous behavior. However, in any given problem it may be difficult to determine a priori the type of behavior that is likely to occur in different parts of the problem domain. For example non-linear and discontinuous behavior may occur only in a small part, whereas the main part behaves elastically. An example of this occurs in underground excavation either in tunneling or mining. The idea proposed and reported here is to let the simulation program automatically determine the areas which need a different type of method and to automatically adapt the mesh during an analysis. The approach is to start with BEM analysis, assuming linear elastic behavior, and then determine the zone where the stress field is such that non-linear or discontinuous behavior is likely to occur (the criterion for discontinuous behavior would be for example the presence of tensile stresses that are higher than the tensile strength of the material). After this it is envisaged that the BEM mesh is changed to include a FEM or DEM mesh in parts that have been identified as being more suited to these models. This is then followed by a coupled analysis. Whereas the coupling of BEM/FEM is well established (Beer et al 2008), the coupling of BEM/DEM is more complex. The reason is that very different solution algorithms are used (implicit versus explicit) and the FEM/BEM work with stresses and strains, whereas the DEM works with particle contact forces. This requires some development of methods to transfer stresses from FEM/BEM to contact forces and the development of suitable coupling algorithms. Here we will present the adaptive strategy, the transfer of stresses to the DEM and the iterative coupling strategy that has been adopted. Test examples are presented to demonstrate the applicability of the developed methodology.

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Key words: simulation methods, boundary element method, finite element method, discrete element method

References

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LA SERENA NUMÉRICA I

Sexto Encuentro de Análisis Numérico de Ecuaciones Diferenciales Parciales

Departamento de Matemáticas, Universidad de La Serena, Diciembre 14–16, 2011

Equivalence between two finite element methods for the eddy current problem*

Alfredo Bermúdez † Bibiana López-Rodríguez ‡

Rodolfo Rodríguez‡ Pilar Salgado†

Abstract

The goal of this talk is to prove that two, in principle different, well-known finite element approximations of the eddy current model are equivalent. The first one concerns a formulation involving the magnetic field in the conductor and the magnetic scalar potential in the dielectric ([2]). The second one solves another formulation of the same problem involving the magnetic field in both, the conductor and the dielectric, and a Lagrange multiplier in the dielectric ([1]). The latter is also shown to be equivalent to a third formulation involving two Lagrange multipliers (also introduced in [1], based on results from [3]), which leads to a well posed linear system.

Key words: Time-harmonic eddy current problems, finite element approximation.

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LA SERENA NUMÉRICA I

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Numerical solution of transient nonlinear axisymmetric eddy current models with hysteresis *

Alfredo Bermúdez † M. Dolores Gómez †

Rodolfo Rodriguez ‡ Pablo Venegas ‡

Abstract

This work deals with the mathematical analysis and the computation of transient electromagnetic fields in nonlinear magnetic media with hysteresis. The result of this work complements the content of [1], where existence of the solution has been proved under fairly general assumptions on the **H**–**B** curve, namely, the nonlinear constitutive relation between the magnetic field **H** and the magnetic induction **B**. In our case, the constitutive relation between **H** and **B** is given by a hysteresis operator, i.e. the values of magnetic field not only depends on the present values of magnetic induction but also on the past history. Like in [1], we assume axisymmetry of the fields and then consider two kinds of boundary conditions. Firstly the magnetic field is given on the boundary (Dirichlet boundary condition). Secondly, the magnetic flux through a meridional plane is given, leading to a non-standard boundary-value problem. For both problems, under suitable assumptions, an existence result is achieved. The technique we use is based on implicit time discretization, a priori estimates and passage to the limit by compactness (see, for instance, [3] and [2]). Finally we consider an application: the numerical computation of eddy current losses in laminated media as those used in transformers or electric machines.

*This research was partially supported by CONICYT fellowship, MECESUP Project UCO0713 and Xunta de Galicia under research project INCITE09 207 047 PR and by Ministerio de Ciencia e Innovación (Spain) under research projects Consolider MATHEMATICA CSD2006-00032 and MTM2008-02483.

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LA SERENA NUMÉRICA I
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Departamento de Matemáticas, Universidad de La Serena, Diciembre 14–16, 2011

Sedimentation of suspensions: new aspects of an old problem*

Raimund Bürger†

Abstract

The sedimentation of suspensions consisting of small solid particles dispersed in a viscous fluid is a fundamental process occurring in nature and many industrial applications. Of particular importance to copper mining in Chile are so-called clarifier-thickeners, which are large-diameter, continuously operated settling tanks in which tailings arising from a flotation process are allowed to settle. On the other hand, applications in mineral processing and other areas frequently involve so-called polydisperse suspensions, in which the solid particles belong to a number N of species differing in size or density. The species usually segregate and form areas of different composition. It is the purpose of this contribution to present recent advances in the development of numerical methods for the simulation of both applications. We start with a general introduction to sedimentation models and then consider a 1-d model of polydisperse sedimentation, which is given by a system of N strongly coupled, nonlinear first-order conservation laws for the N local solids volume fraction of the solids species [1, 2]. Solutions of this system are usually discontinuous with kinematic shocks separating areas of different composition. It is demonstrated how the so-called secular equation (can be utilized to estimate the region of hyperbolicity and to extract the eigenstructure of the Jacobian with acceptable effort. This information can be employed for the implementation of weighted essentially non-oscillatory (WENO) schemes for the accurate numerical approximation of the model. Numerical examples are presented. Multidimensional versions of the same model involve the necessity to solve additional equations for the motion of the mixture, e.g. the Stokes system with a concentration-dependent viscosity and forcing term. We demonstrate how multiresolution methods may be applied to the numerical solution of such coupled problems for the case of settling of suspensions in inclined channels [4]. Finally, we consider a multi-dimensional sedimentation-consolidation model in clarifier-thickener units [3]. The model is given by a parabolic equation describing the evolution of the local solids concentration coupled with a version of the Stokes system for an incompressible fluid. In cylindrical coordinates, and if an axially symmetric solution is assumed, the original problem reduces to two space dimensions. A novel finite volume element method is introduced for the spatial discretization, where the velocity

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field and the solids concentration are discretized on two different dual meshes. Numerical experiments illustrate properties of the model and the satisfactory performance of the proposed method. This presentation is based on joint work with Rosa Donat and Pep Mulet (Valencia, Spain), Carlos A. Vega (Barranquilla, Colombia), Ricardo Ruiz-Baier (Lausanne, Switzerland), Kai Schneider (Marseille, France) and Héctor Torres (La Serena, Chile).

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LA SERENA NUMÉRICA I

Sexto Encuentro de Análisis Numérico de Ecuaciones Diferenciales Parciales

Departamento de Matemáticas, Universidad de La Serena, Diciembre 14–16, 2011

A clarifier-thickener model including flocculant transport and adsorption

Raimund Bürger* Kenneth H. Karlsen†

Héctor Torres‡ John D. Towers§

Abstract

We consider a one-dimensional system of conservation laws modeling clarifier-thickener units for flocculated suspensions. The novelty of this model is included in the process of settling two equations that model the transport and adsorption of the flocculant in the mixture. Additionally, we also have a reactive term on the right hand side of the adsorption and transport equations. For us this term describes the adsorption (and desorption) of the flocculant in the mixture. We consider that the adsorbed flocculant appears as a parameter in the nonlinear functions describing the solid-fluid relative velocity, then the available and adsorbed flocculant are not just scalars that travel with the fluid and the solid. A numerical algorithm for the solution of this system is presented along with numerical examples.

Key words: Conservation law, Discontinuous flux, Clarifier-thickener, Sedimentation.

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LA SERENA NUMÉRICA I
Sexto Encuentro de Análisis Numérico de Ecuaciones Diferenciales Parciales
Departamento de Matemáticas, Universidad de La Serena, Diciembre 14–16, 2011

Implicit-Explicit Runge–Kutta schemes and finite elements with symmetric stabilization for advection-diffusion equations

Erik Burman* Alexandre Ern†

Abstract

We analyze a two-stage explicit-implicit Runge–Kutta scheme for time discretization of advection-diffusion equations. Space discretization uses continuous, piecewise affine finite elements with interelement gradient jump penalty; discontinuous Galerkin methods can be considered as well. The advective and stabilization operators are treated explicitly, whereas the diffusion operator is treated implicitly. Our analysis hinges on L^2-energy estimates on discrete functions in physical space. Our main results are stability and quasi-optimal error estimates for smooth solutions under a standard hyperbolic CFL restriction on the time step, both in the advection-dominated and in the diffusion-dominated regimes. The theory is illustrated by numerical examples.

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Stability and convergence for a complete model of mass diffusion

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Abstract

We propose a fully discrete scheme for approximating a three-dimensional, strongly nonlinear model of mass diffusion, also called the complete Kazhikhov-Smagulov model. The scheme uses a C^0 finite element approximation for all unknowns (density, velocity and pressure), even though the density limit, solution of the continuous problem, belongs to H^2. A first order time discretization is used such that, at each time step, one only needs to solve two decoupled linear problems for the discrete density and the velocity-pressure, separately. We extend to the complete model, some stability and convergence results already obtained by the last two authors for a simplified model where l^2-terms are not considered, l being the mass diffusion coefficient. Now, different arguments must be introduced, based mainly on an induction process with respect to the time step, obtaining at the same time the three main properties of the scheme: an approximate discrete maximum principle for the density, weak estimates for the velocity and strong ones for the density. Furthermore, the convergence towards a weak solution of the density-dependent Navier-Stokes problem is also obtained as $l \to 0$ (jointly with the space and time parameters). Finally, some numerical computations prove the practical usefulness of the scheme.

Key words: three-dimensional Kazhikhov-Smagulov model, density-dependent Navier-Stokes problem, finite elements, stability, convergence.

Mathematics subject classifications (1991): 35Q35, 65M12, 65M60.

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LA SERENA NUMÉRICA I
Sexto Encuentro de Análisis Numérico de Ecuaciones Diferenciales Parciales
Departamento de Matemáticas, Universidad de La Serena, Diciembre 14–16, 2011

Solving Dirichlet boundary-value problems on general domains by extensions from subdomains

Bernardo Cockburn* Manuel Solano†

Abstract

We present a technique for numerically solving Dirichlet boundary-value problems on a general domain Ω. We do not assume Ω polygonal. This is achieved by using suitably defined extensions from polyhedral subdomains $\mathsf{D}_h \subset \Omega$; the problem of dealing with curved boundaries is thus reduced to the evaluations of simple line integrals. The technique is independent of the representation of the boundary and of the space dimension. Moreover, it allows the use of only polyhedral elements and high order approximations. In the polyhedral subdomains D_h, we use a hybridizable discontinuous Galerkin method ([1]). We apply this technique to pure-diffusion ([2]) and convection-diffusion ([3]) problems and provide numerical experiments showing that the convergence properties of the resulting method are the same as those for the case in which $\Omega = \mathsf{D}_h$ whenever the distance of D_h to $\partial\Omega$ is of order h.

Key words: curved domains, immersed boundary methods, discontinuous Galerkin methods

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LA SERENA NUMÉRICA I

Sexto Encuentro de Análisis Numérico de Ecuaciones Diferenciales Parciales

Departamento de Matemáticas, Universidad de La Serena, Diciembre 14–16, 2011

An update of the DPG method for wave propagation problems*

Leszek Demkowicz† Jay Gopalakrishnan‡ Ignacio Muga§

Abstract

The discontinuous Petrov-Galerkin (DPG) method has shown outstanding numerical results for high frequency wave propagation and other singular perturbation problems [3, 2]. Indeed, for acoustic waves, we find that the method exhibits negligible phase errors (otherwise known as pollution errors) even in the lowest order case. This is despite the negative result of Babuška & Sauter [1] predicting that in two dimensions, it is impossible to eliminate the pollution effect completely. Theoretically, we are able to prove error estimates that explicitly show the dependencies with respect to the wavenumber ω, the mesh size h, and the polynomial degree p. But the current state of the theory does not fully explain the remarkably good numerical phase errors. In this talk, we give an update of the state of the art of our research in this subject.

Key words: discontinuous Petrov Galerkin, Helmholtz equation, phase error

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LA SERENA NUMÉRICA I

Sexto Encuentro de Análisis Numérico de Ecuaciones Diferenciales Parciales

Departamento de Matemáticas, Universidad de La Serena, Diciembre 14–16, 2011

Robust DPG method for convection-dominated diffusion problems *

Leszek Demkowicz† Norbert Heuer ‡

Abstract

We propose and analyze a DPG method for convection-dominated diffusion problems which provides robust L^2 error estimates for the field variables, and which are quasi-optimal in the energy norm. Key feature of the method is to construct test functions defined by a variational formulation with bilinear form (test norm) specifically designed for the goal of robustness. Main theoretical ingredient is a stability analysis of the adjoint problem. Numerical experiments underline our theoretical results and, in particular, confirm robustness of the DPG method for well-chosen test norms.

Key words: convection-dominated diffusion, *hp*-adaptivity, discontinuous Petrov Galerkin method

Mathematics subject classifications (1991): 65N30, 35L15

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LA SERENA NUMÉRICA I

Sexto Encuentro de Análisis Numérico de Ecuaciones Diferenciales Parciales

Departamento de Matemáticas, Universidad de La Serena, Diciembre 14–16, 2011

A priori error analysis of a fully-mixed finite element method for a two-dimensional fluid-solid interaction problem *

Carolina Domínguez† Gabriel N. Gatica‡
Salim Meddahi§ Ricardo Oyarzúa¶

Abstract

We introduce and analyze a fully-mixed finite element method for a fluid-solid interaction problem in 2D. The model consists of an elastic body which is subject to a given incident wave that travels in the fluid surrounding it. Actually, the fluid is supposed to occupy an annular region, and hence a Robin boundary condition imitating the behavior of the scattered field at infinity is imposed on its exterior boundary, which is located far from the obstacle. The media are governed by the elastodynamic and acoustic equations in time-harmonic regime, respectively, and the transmission conditions are given by the equilibrium of forces and the equality of the corresponding normal displacements. We first apply dual-mixed approaches in both domains, and then employ the governing equations to eliminate the displacement u of the solid and the pressure p of the fluid. In addition, since both transmission conditions become essential, they are enforced weakly by means of two suitable Lagrange multipliers. As a consequence, the Cauchy stress tensor and the rotation of the solid, together with the gradient of p and the traces of u and p on the boundary of the fluid, constitute the unknowns of the coupled problem. Next, we show that suitable decompositions of the spaces to which the stress and the gradient of p belong, allow the application of the Babuska-Brezzi theory and the Fredholm alternative for analyzing the solvability of the resulting continuous formulation. The unknowns of the solid and the fluid are then approximated by a conforming Galerkin scheme defined in terms of PEERS elements in the solid, Raviart-Thomas of lowest order in the fluid, and continuous piecewise linear functions on the boundary. Then, the analysis of the discrete method relies on a stable decomposition of the corresponding finite element spaces and also on a classical result on projection methods

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for Fredholm operators of index zero. Finally, some numerical results illustrating the theory are presented.

Key words: mixed finite elements, Helmholtz equation, elastodynamic equation

Mathematics subject classifications (1991): 65N30, 65N12, 65N15, 74F10, 74B05, 35J05

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LA SERENA NUMÉRICA I
Sexto Encuentro de Análisis Numérico de Ecuaciones Diferenciales Parciales
Departamento de Matemáticas, Universidad de La Serena, Diciembre 14–16, 2011

Finite element and boundary element coupling for fluid-structure interaction *

CATALINA DOMÍNGUEZ† E. P. STEPHAN‡ M. MAISCHAK§

Abstract

We consider the coupling of finite elements and boundary elements to solve a fluid structure interaction problem. We consider a time-harmonic vibration and scattering problem for homogeneous, isotropic, elastic solids surrounded by a compressible, inviscid and homogeneous fluid. These methods combine integral equations for the exterior fluid and finite element methods for the elastic structure. The eigenvalues of the interior Helmholtz problem induce non-unique solutions of the integral equations. Therefore we focus on two stable variational formulations, a symmetric and a non-symmetric formulation. These formulations are stable in the sense that they now provide a unique solution. For both stable formulations we derive a posteriori error estimates, a residual error estimator and a hierarchical error estimator. From the error estimators we compute local error indicators which allow us to develop an adaptive mesh refinement strategy. We present the numerical results for the 2D and 3D cases.

Key words: Fluid structure interaction problem. FE/BE coupling method, Galerkin method, a posteriori error estimator, residual error estimator, two-level hierarchical error estimator, adaptive algorithm.

Mathematics subject classifications (1991): 65N30, 65N15, 74F10, 76Q05.

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LA SERENA NUMÉRICA I
Sexto Encuentro de Análisis Numérico de Ecuaciones Diferenciales Parciales
Departamento de Matemáticas, Universidad de La Serena, Diciembre 14–16, 2011

Tridiagonal preconditioning for Poisson-like difference equations with flat grids: Application to incompressible atmospheric flow problem

Melitta Fiebig-Wittmaack* Wolfgang Boersch-Supan†
Ingeborg Bischoff-Gauss‡ Orlando Astudillo§

Abstract
The convergence of many iterative procedures, in particular of the conjugate gradient method, strongly depends on the condition number of the linear system to be solved. In cases with a large condition number, therefore, preconditioning is often used to transform the system into an equivalent one, with a smaller condition number and therefore faster convergence. For Poisson-like difference equations with flat grids, the vertical part of the difference operator is dominant and tridiagonal and can be used for preconditioning. Such a procedure has been applied on incompressible atmospheric flows to maintain incompressibility, where a system of Poisson-like difference equations is to be solved for the dynamic pressure part. In the mesoscale atmospheric model KAMM, convergence has been speeded up considerably by tridiagonal preconditioning, even though the system matrix is not symmetric and, hence, the biconjugate gradient method must be used.

Key words: Poisson-like equation; condition number; preconditioning; convergence acceleration; atmospheric model; flat grids.

Mathematics subject classifications (1991): 65N22;65F10;86A10;76D05

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LA SERENA NUMÉRICA I
Sexto Encuentro de Análisis Numérico de Ecuaciones Diferenciales Parciales
Departamento de Matemáticas, Universidad de La Serena, Diciembre 14–16, 2011

Greedy approximation of a high-dimensional PDE arising from kinetic theory models of dilute polymers*

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Abstract

We investigate the convergence of a nonlinear approximation method introduced by Ammar et al. for the numerical solution of high-dimensional Fokker–Planck equations featuring in Navier–Stokes–Fokker–Planck systems that arise in kinetic models of dilute polymers. In the case of Poisson's equation on a rectangular domain in $\mathbb{R}^2$, subject to a homogeneous Dirichlet boundary condition, the mathematical analysis of the algorithm was carried out recently by Le Bris, Lelièvre and Maday by exploiting its connection to greedy algorithms from nonlinear approximation theory explored, for example, by DeVore and Temlyakov; hence, the variational version of the algorithm, based on the minimization of a sequence of Dirichlet energies, was shown to converge. We extend the convergence analysis of the pure greedy and orthogonal greedy algorithms considered by Le Bris, Lelièvre and Maday to the technically more complicated case where the Laplace operator is replaced by a high-dimensional Ornstein–Uhlenbeck operator with unbounded drift, of the kind that appears in Fokker–Planck equations that arise in bead-spring chain type kinetic polymer models with finitely extensible nonlinear elastic potentials, posed on a high-dimensional Cartesian product configuration space $\mathsf{D} = D_1 \times \cdots \times D_N$ contained in $\mathbb{R}^{Nd}$, where each set D_i, $i = 1, \dots, N$, is a bounded open ball in $\mathbb{R}^d$, $d = 2, 3$.

Key words: Nonlinear approximation, greedy algorithm, Fokker–Planck equation

2000 Mathematics Subject Classification: 65N15, 65D15, 41A63, 41A25

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Numerical solution of an equation describing the centrifugal settling with coalescence of polydisperse liquid-liquid dispersions using the fixed pivot technique *

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Abstract

The centrifugal settling and coalescence of a polydisperse dispersion of two inmiscble liquids with a continuous droplet size distribution in a rotating tube or basket centrifuge can be modeled by a integro-partial differential equation (IPDE), which is an extension of a previous spatially distributed population balance equation describing gravity settling and coalescence of such a mixture. This IPDE is projected onto a system of convective dominant partial differential equations by discretizing the droplet diameter. This is accomplished by using the fixed-pivot technique of Kumar and Ramkrishna (Chem. Eng. Sci. 51 (1996a) 1311–1332) handling any two integral properties of the population number density. The resulting system of PDEs is split into two systems, of homogeneous PDEs and ODEs. The homogeneous PDEs and the ODEs are discretized using the second-order non-oscillatory central differencing scheme of Kurganov and Tadmor (J. Comput. Phys. 160 (2000) 241–282) and the second-order two-stage Runge-Kutta method, respectively. Simulations are presented, illustrating the coalescence and the formation of sediment of the disperse phase, and the effect of various centrifuge geometries for both cases, when the disperse phase (droplets) is less dense than the continuous phase and viceversa. In particular, the model predicts the radial variation of the composition of the disperse phase layer forming at the inner or outer wall.

Key words: centrifugal settling, coalescence, polydisperse dispersion, population balance equation

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Sexto Encuentro de Análisis Numérico de Ecuaciones Diferenciales Parciales
Departamento de Matemáticas, Universidad de La Serena, Diciembre 14–16, 2011

Analysis of an augmented fully-mixed finite element method for a three-dimensional fluid-solid interaction problem*

Gabriel N. Gatica† Antonio Márquez‡ Salim Meddahi§

Abstract

We introduce and analyze an augmented fully-mixed finite element method for a fluid-solid interaction problem in 3D. The media are governed by the acoustic and elastodynamic equations in time-harmonic regime, and the transmission conditions are given by the equilibrium of forces and the equality of the corresponding normal displacements. We first employ dual-mixed variational formulations in both domains, which yields the Cauchy stress tensor and the rotation of the solid, together with the gradient of the pressure of the fluid, as the preliminary unknowns. This approach allows us to extend an idea from a recent own work in such a way that both transmission conditions are incorporated now into the definitions of the continuous spaces, and therefore no unknowns on the coupling boundary appear. As a consequence, the pressure of the fluid and the displacement of the solid become explicit unknowns of the coupled problem, and hence two redundant variational terms arising from the constitutive equations, both of them multiplied by stabilization parameters, need to be added for well-posedness. In fact, we show that explicit choices of the above mentioned parameters and a suitable decomposition of the spaces allow the application of the Babuška-Brezzi theory and the Fredholm alternative for concluding the solvability of the resulting augmented formulation. The unknowns of the fluid and the solid are then approximated by a conforming Galerkin scheme defined in terms of Arnold-Falk-Winther and Lagrange finite element subspaces of order 1. The analysis of the discrete method relies on a stable decomposition of the finite element spaces and also on a classical result on projection methods for Fredholm operators of index zero.

Key words: mixed finite elements, Helmholtz equation, elastodynamic equation

Mathematics subject classifications (1991): 65N30, 65N12, 65N15, 74F10, 74B05, 35J05

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LA SERENA NUMÉRICA I
Sexto Encuentro de Análisis Numérico de Ecuaciones Diferenciales Parciales
Departamento de Matemáticas, Universidad de La Serena, Diciembre 14–16, 2011

On numerical schemes for a Cahn Hilliard diffuse interface model*

Francisco Guillén González   Giordano Tierra†

Abstract

The Cahn-Hilliard model was originally introduced in [5] and describes the complicated phase separation and coarsening phenomena in the mixture of different fluids, solid or gas where only two different concentration phases can exist stably. Let $\Omega \in \mathbb{R}^d, d = 2, 3$ a bounded domain with boundary $\partial\Omega$. The model reads,

$$\begin{cases} \phi_t = \Delta w,\, w = -\Delta\phi + \dfrac{1}{\varepsilon^2}F'(\phi) & \text{in } \Omega \times (0,T) \\ \dfrac{\partial\phi}{\partial n} = 0,\, \dfrac{\partial w}{\partial n} = 0 & \text{in } \partial\Omega \end{cases}$$

where ϕ represents the phase field function, $F(\phi)$ is a double well potential and ε is a small parameter known as 'interaction length'. The following energy law holds

$$E(\phi) = \int_\Omega \left(\frac{1}{2}|\nabla\phi|^2 + F(\phi) \right) dx.$$

Numerical schemes to approximate the Cahn-Hilliard equation have been widely studied in recent times due to its connection with many physically motivated problems. In this work we propose two different ways to approximate the double-well potential term, driving to two new linear schemes. The first one is optimal from the numerical dissipation point of view meanwhile the second one allows us to design unconditionally stable linear schemes. We present first and second order in time linear schemes to approximate this problem, detailing their advantages over other linear schemes that have been previously introduced in the literature. Furthermore, we compare all the schemes through several computational experiments.

Key words: diffuse interface phase-field, Cahn-Hilliard, mixed finite element, long time stability.

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LA SERENA NUMÉRICA I

Sexto Encuentro de Análisis Numérico de Ecuaciones Diferenciales Parciales

Departamento de Matemáticas, Universidad de La Serena, Diciembre 14–16, 2011

Operator preconditioning for two-dimensional screen and fracture problems using boundary elements

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Abstract

Calderón preconditioning has been successfully used in the past for solving boundary integrals over surfaces without boundaries [2, 1]. The situation changes drastically when considering open boundaries as when modeling screens or cracks. Indeed, Calderón identities break down due to the disappearance of the double layer boundary operator (and its adjoint). The associated weakly singular and hypersingular operators no longer map fractional Sobolev spaces in a dual fashion but degenerate into different subspaces depending on their extensibility by zero. Recently, Jerez-Hanckes and Nédélec [3], showed that a jump and average decomposition of the volume solution leads to precise coercivity results in fractional Sobolev spaces and characterize the mismatch occurring between associated functional spaces in the aforementioned limiting cases. Moreover, they present explicit and exact variational formulations when considering an open interval as well as for their corresponding inverses and naturally define Calderón-type identities in each case with potential use as preconditioners. In this presentation, we show first numerical implementations of these preconditioners and discuss future extensions.

Key words: operator preconditioning, boundary element methods, screen problems

Mathematics subject classifications (1991): 45P05, 65N38, 31A10, 46E35

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LA SERENA NUMÉRICA I

Sexto Encuentro de Análisis Numérico de Ecuaciones Diferenciales Parciales

Departamento de Matemáticas, Universidad de La Serena, Diciembre 14–16, 2011

A strong finite element coupling of fluid flow with porous media flow

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Abstract

Our aim is to propose a new finite element scheme for the coupling of fluid flow with porous media flow. In the approach in which the Darcy problem is set in its natural $H(\mathrm{div})$ formulation (cf. [1] and the references therein) and the Stokes problem is expressed in velocity-pressure form, the transmission condition ensuring global mass conservation is handled weakly through a Lagrange multiplier representing the pressure on the coupling interface Σ. This strategy requires two finite element meshes on the coupling boundary Σ satisfying a stability condition between their corresponding mesh sizes. Such a restriction is quite cumbersome in 3D computations. We propose here a formulation in which the balance on Σ between the Darcy flux and the normal component of the fluid velocity is imposed exactly (strongly) at the discrete level. We discuss the kind of conditions the discrete spaces for Stokes and Darcy have to satisfy to deliver stable methods for the global formulation.

Key words: mixed finite elements, Stokes problem, Darcy problem

Mathematics subject classifications (1991): 65N30, 65N12, 65N15, 74F10, 74B05, 35J05

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LA SERENA NUMÉRICA I
Sexto Encuentro de Análisis Numérico de Ecuaciones Diferenciales Parciales
Departamento de Matemáticas, Universidad de La Serena, Diciembre 14–16, 2011

On mathematical models and FVM simulations for convective solidification industrial processes*

Nelson Moraga† Mauricio Godoy† Carlos Garrido†
Leopoldo Jauriat‡, Roberto Lemus† ‡

Abstract

Mathematical models and numerical results of unsteady temperature and velocity distributions, calculated using the Finite Volume Method, for liquid to solid phase change with natural convection are discussed. Typical cases found in industrial applications are presented for food freezing, ternary alloy solidification and binary alloy cylindrical solidification inside moulds. Problem 1 predicts either a solid food or a non-Newtonian liquid food freezing in air inside a freezer, with a conjugate natural convection/difusion or convective mathematical model. Problem 2 describes a ternary alloy solidification process in a square mould, using a porous model and a temperature dependent liquid fraction, in which the sequential solution of the discrete systems of unsteady, non-linear, coupled, partial differential equations is obtained with a new sequential algorithm, PSIMPLER. Problem 3 predicts the solidification of a binary alloy, Al-1.7wt%Si, in a cylindrical horizontal mould, based on a non–Newtonian power law model for the relation between shear stress and deformation rate, [1].

Key words: Finite Volume Method, PSIMPLER algorithm

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A particular mixed formulation for interface approximation of Darcy flow in a narrow channel *

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Abstract

A particular mixed formulation is introduced for the singular problem of Darcy flow in a porous medium in a region containing a narrow fracture or channel with width $\mathcal{O}(\epsilon)$ and high permeability $\mathcal{O}(1/\epsilon)$. The formulation allows to introduce more general transmission conditions for fluid exchange between the domains than those allowed by the L^2-H^1 and the $H(\mathrm{div})$-L^2 mixed formulations. For a channel defined by two vertical translates of a piecewise C^2 surface the solution converges as $\epsilon \to 0$ to that of Darcy flow coupled to tangential flow on the lower-dimensional interface or boundary. Numerical experiments will be presented to illustrate aspects such as convergence, stability and implementation of the finite element method for the interaction between a regular domain in $\mathbb{R}^N$ and a lower-dimensional manifold.

Key words: porous media, heterogenous, fissures, coupled Darcy systems.

Mathematics subject classifications (1991): Primary 35K50, 35B25; Secondary 80A20, 35F15

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LA SERENA NUMÉRICA I
Sexto Encuentro de Análisis Numérico de Ecuaciones Diferenciales Parciales
Departamento de Matemáticas, Universidad de La Serena, Diciembre 14–16, 2011

Analysis of a modified discontinuous Galerkin recovery scheme for diffusion problems

Mauricio Osorio* Donald French† Marshall Galbraith‡

Abstract

A theoretical error analysis using standard Sobolev space energy arguments is furnished for a class of discontinuous Galerkin (DG) schemes that are modified versions of one of those introduced by van Leer and Nomura. These schemes, which use discontinuous piecewise polynomials of degree q, are applied to a family of one-dimensional elliptic boundary value problems. The modifications to the original method include definition of a recovery flux function via a symmetric L^2-projection and the addition of a penalty or stabilization term. The method is found to have a convergence rate of $O(h^q)$ for the approximation of the first derivative and $O(h^{q+1})$ for the solution. Computational results for the original and modified DG recovery schemes are provided contrasting them as far as complexity and cost. Numerical examples are given which exhibit sub-optimal convergence rates when the stabilization terms are omitted.

Key words: Error Analysis, Discontinuous Galerkin, Recovery Scheme

Mathematics subject classifications (1991): 65N12, 65N30

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LA SERENA NUMÉRICA I
Sexto Encuentro de Análisis Numérico de Ecuaciones Diferenciales Parciales
Departamento de Matemáticas, Universidad de La Serena, Diciembre 14–16, 2011

Analysis of a cartesian PML approximation to acoustic scattering problems in $\mathbb{R}^n$

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Abstract

We consider the application of a perfectly matched layer (PML) technique in Cartesian geometry to approximate solutions of the acoustic scattering problem in the frequency domain. Our goal is to develop stability and exponential convergence for PML approximations on truncated (bounded) domains. To illustrate the issues, we start by examining the analysis for a simple PML problem in one spatial dimension. We then consider some of the issues in developing analogous stability properties for multidimensional PML approximations. In particular, we consider the case when the PML stretching is applied independently in each coordinate direction, the so-called Cartesian PML. The compact perturbation arguments used in the one dimensional problem fail in higher dimensions. Two distinct approaches circumventing this problem will be described. The first is based on locating the essential spectrum of the PML operator. The second involves the development of a solution operator for the PML equations in terms of an integral transform. The second approach is particularly interesting as it leads to estimates which show that stability of the truncated PML problem can be achieved provided that the product of the size of the truncated domain times the PML strength is sufficiently large. This means that stability can be achieved with a fixed sized computational domain in conjunction with a large PML coefficient. Finally, the results of computational experiments illustrating the theory will be given and extensions to other problems will be discussed.

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Numerical analysis of a Stokes' approximation for hydrodynamic interactions of droplets in a turbulent tropical cloud *

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Abstract

The numerical simulation of tropical clouds is a challenging problem requiring the numerical solution of the Navier-Stokes equations. In this work we focus on the hydrodynamic interactions of droplets. The typical droplet size is smaller than the smallest length scale in the turbulent simulation so the Stokes' equation is used [9, 10, 1]. The Stokes' approximation requires a solution of a linear system of equation every time the droplets move, thus a fast and accurate method is needed. The linear system is solved by GMRes [4, 5, 8] and a Restricted Schwarz preconditioner [2]. We explore the convergence properties of the sequence of linear system. A perturbation analysis reveals key features of why GMRes is so effective in this case [3, 5, 6]. We finally discuss a parallel implementation on massively parallel peta-scale machines [7] and a possible extension to use a uniformly valid approximation for the interactions.

Key words: Stokes' approximation, convergence of GMRes, parallel computing

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Finite volume schemes of high order for flows in different layers of unsaturated porous media*

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Abstract

Finite volume schemes of arbitrary order of accuracy in time and space for solving the flow in unsaturated porous media are presented. A high order WENO reconstruction procedure is applied to the cell averages at the current time level. The temporal evolution of the reconstruction polynomials is computed globally using the governing equations. A *global* space–time discontinous Galerkin (GDG) finite element scheme is used to resolve the small gradients produced by the flows in unsatured porous media. An iterative solution of the matrix procedure is used to solve the system of equation that results from the global scheme proposed. Finally an explicit space–time integration over each control volume, using the GDG solution at the Gaussian integration point allows the calculation of the intercell fluxes. The convergence of the new scheme developed is tested with the exact transient diffusion–reaction solution. Three test cases are presented for comparing the accuracy of our ADER-FV method of second to fourth order solution in unsaturated layered soil.

Key words: unsatured porous media, high–order schemes, finite volume methods, Richards equations, ADER schemes, space–time discontinous Galerkin.

Mathematics subject classifications (1991): 65N30, 65N12, 65N15, 74F10, 74B05, 35J05

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